

Criticality and Scaling in 4D Quantum Gravity^{*}

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Abstract

We present a simple argument which determines the critical value of the anomaly coefficient in four dimensional conformal factor quantum gravity, at which a phase transition between a smooth and elongated phase should occur. The argument is based on the contribution of singular configurations (“spikes”) which dominate the partition function in the infrared. The critical value is the analog of $c = 1$ in the theory of random surfaces, and the phase transition is similar to the Berezinskii-Kosterlitz-Thouless transition. The critical value we obtain is in agreement with the previous canonical analysis of physical states of the conformal factor and may explain why a smooth phase of quantum gravity has not yet been observed in simplicial simulations. We also rederive the scaling relations in the smooth phase in light of this determination of the critical coupling.

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In recent work we have developed a framework which permits the study of the effects of gravitational fluctuations at large distances [1]–[5]. It is based on the quantization of a low energy effective action for the spin-0 or conformal part of the metric, which is determined by the trace anomaly. This action implies the existence of an infrared stable fixed point for quantum gravity in four dimensions with nontrivial scaling behavior [1]. On the other hand, quantum gravity can be reformulated as a statistical model on a random lattice and studied numerically [6]. There is then the attractive possibility for testing the continuum predictions of an infrared fixed point by quantitative measurements in the lattice approach [4, 7].

In a previous letter [4] we discussed the scaling relations for the partition function and observables in the conformal phase, based on the results of the infrared fixed point behavior found earlier. Since that time we have performed an extensive canonical analysis of the constraints of diffeomorphism invariance on the Einstein universe $R \times S^3$, and found a discrete spectrum of physical states in the Fock space which survive the imposition of the constraints [5]. Each of these physical states corresponds to an operator of scaling dimension or conformal weight 4 constructed from the conformal part of the metric and its derivatives. The existence of these states in a pure gravity theory without matter is in sharp contrast to the analogous quantization of the Liouville theory on the cylinder $R \times S^1$ where there is only one state (the “vacuum”) and only one operator (the identity or volume operator). In four dimensions, there is a tower of operators describing marginal deformations of the theory away from its infrared fixed point. In the semi-classical limit, the two lowest of these become the Einstein-Hilbert and cosmological (or volume) terms. Each of these two operators comes with its own scaling behavior and anomalous dimensions, and each of them seems to imply a different value of the coupling where a phase transition to a highly nonclassical (or branched polymer) phase of quantum gravity could occur. Since the situation is more complicated than the $D = 2$ case, the scaling behavior

of observables should be reconsidered from the present vantage point of a more complete understanding of the physical states and the operators that create them, and the critical coupling determined by the point at which the scaling exponents first cease to be real.

In two dimensions there is a simple argument for the critical value $c = 1$ of the embedding dimension in Polyakov’s theory of random surfaces (or noncritical bosonic string theory) [8]. The argument is reminiscent of the Berezenskii-Kosterlitz-Thouless (BKT) argument for a phase transition in the 2D x-y model [9], in that it relies upon constructing a singular solution to the classical equations (called a “spike” in this context), which makes a contribution to the action that depends logarithmically on the infrared cut-off. Since the entropy of such configurations also grows logarithmically with the volume of the system, there is a critical value of the coupling at which the entropy always overwhelms the action and the partition function is dominated by a dense gas of such singular spikes. Conversely, if the coupling is adjusted in the opposite direction then such configurations are always suppressed in the infinite volume limit, and the geometry of the surface can be reasonably smooth. Since the coupling is also the coefficient of the trace anomaly in 2D, this argument also tells us what the critical value of the matter central charge is, and why one should expect a phase transition from a smooth to a branched polymer phase at this critical coupling of $c = 1$. This phase transition has been verified in the dynamical triangulation approach to 2D random surfaces [10].

Since a branched polymer (or elongated) phase has been observed also in 4D simplicial quantum gravity [6, 7], it is quite natural to suspect that an analogous argument involving the “liberation of spikes” in the subcritical region of a coupling constant (Q^2 defined below) should apply again in this case, as remarked in a recent paper [11]. In this note we develop this idea that the “half-wormhole” or “spike” configurations are the relevant ones for the phase transition at a certain critical value of the anomaly coefficient and we study the implications for 4D simplicial simulations and scaling relations in the smooth phase.

The Critical Coupling. In four dimensions, the analog of the Polyakov-Liouville theory of random surfaces is obtained by considering the effective action for the conformal factor of the metric induced by the 4D trace anomaly. This effective action takes the form [1]

$$S_E[\sigma] = \frac{Q^2}{(4\pi)^2} \int d^4x \sqrt{g} [\sigma \Delta_4 \sigma + \frac{1}{2}(G - \frac{2}{3} \square R) \sigma] , \quad (1)$$

where Δ_4 is the unique Weyl covariant fourth order differential operator on scalars, $e^{2\sigma}$ is the conformal factor of the metric (taken here with Euclidean signature), and Q^2 is the coefficient of the Gauss-Bonnet term G in the trace anomaly. It is normalized such that [2]

$$Q^2 = \frac{1}{180} (N_S + \frac{11}{2} N_{WF} + 62 N_V - 28) + Q_{grav}^2 , \quad (2)$$

where N_S, N_{WF}, N_V are the number of free scalars, Weyl fermions and vector fields and Q_{grav}^2 is the contribution of spin-2 gravitons, which has not yet been determined unambiguously. The -28 contribution is that of the σ field itself which is the only known negative contribution to Q^2 . Thus, it is Q^2 which plays the role analogous to matter central charge and Δ_4 which plays the role of the kinetic operator \square in $D = 2$. In flat background coordinates $\Delta_4 = \square^2$.

Now we make the following observation. Because of the fourth order conformal differential operator matched to the number of dimensions, the propagator of this operator is a logarithm, just as is that of \square in $D = 2$. This means that we have a situation analogous to that in 2D. Although the Mermin-Wagner theorem forbids a spontaneously broken phase with massless excitations, the 2D x-y model does exhibit a BKT phase transition at a certain critical temperature (*i.e.* coupling in the 2D Euclidean field theory), which is just the result of the logarithmic growth of the massless conformal invariant \square^{-1} propagator in $D = 2$. Because the conformal propagator Δ_4^{-1} in four dimensions has logarithmic growth at large distances, one should expect a BKT-like phase transition at a certain critical coupling Q_{cr}^2 in 4D for essentially the same reason as in the original x-y model or in the 2D theory of random surfaces.

Indeed, we can construct exactly the same spike solution in $D = 4$ as in $D = 2$,

$$\sigma_S(x) = q \ln |x - x_0| \quad (3)$$

with exactly the same interpretation. It is the solution of the classical equation following from (1) with a delta function singularity at $x = x_0$ of strength q . In order to regulate the singular behavior of this configuration one may introduce an ultraviolet (UV) cut-off a , replacing $|x - x_0|^2$ by $1 + |x - x_0|^2/a^2$ in the logarithm. This UV cut-off will be of order of the lattice spacing in the simplicial simulations. The infrared (IR) cut-off of the logarithm will be provided by the finite volume.

Evaluating the action S_E on this spike configuration in a large but finite spherical volume of radius L we find

$$S_E[\sigma_S] = \frac{1}{2} Q^2 q^2 \ln \left(\frac{L}{a} \right) . \quad (4)$$

On the other hand the number of ways of placing this configuration in the volume V is proportional to the volume, $V \sim L^4$. Hence, the entropy grows like $4 \ln(\frac{L}{a})$ and the free energy of such configurations behaves like

$$F[\sigma_S] = \left(\frac{1}{2} Q^2 q^2 - 4 \right) \ln \left(\frac{L}{a} \right) \quad (5)$$

for large L . It follows that for $Q^2 > 8/q^2$ these singular configurations will have positive free energy and be suppressed in the thermodynamic limit $L \rightarrow \infty$, while if the inequality is reversed we must expect them to dominate the partition function. Consequently, there is a critical value of the coupling Q^2 at which we expect to see a phase transition from a phase with many sharp elongated spikes to one where such singular configurations are suppressed. The analogy to the BKT argument involving vortices in the x-y phase field is obvious.

In two dimensions, Cates argued that the value $q = -1$ (in the present notation) is special because this is the strength at which the singularity is first strong enough to give a

divergent contribution to the classical volume (area in $D = 2$) as the cut-off a is removed [8]. Hence, these are the configurations with the lowest free energy that can dominate the partition function in the continuum limit $L \gg a$. Since the volume or cosmological operator is a marginal deformation of the free Liouville theory, this argument for $q = -1$ can be turned into a renormalization group analysis in the IR as well, by taking into account the gravitational “dressing” of the volume operator due to the loop corrections of the free \square^{-1} propagator near its Gaussian fixed point. Then we find that $q = -1$ is precisely the condition that justifies the neglect of the cosmological term and the use of the spike solution to the free Liouville theory in the infinite volume limit $L \rightarrow \infty$. Let us give this argument why $q = -1$ is the relevant spike solution in four dimensions as well.

In four dimensions, there are two terms with fewer derivatives that can be added to the free action (1). They are the volume or cosmological term,

$$\lambda S_0[\sigma] = \lambda \int d^4x \sqrt{g} = \lambda \int d^4x e^{4\sigma} \rightarrow \lambda \int d^4x e^{\beta_0 \sigma}. \quad (6)$$

and the Einstein-Hilbert term,

$$\begin{aligned} \frac{1}{2\kappa} S_2[\sigma] &= -\frac{1}{2\kappa} \int d^4x \sqrt{g} R = \frac{3}{\kappa} \int d^4x e^{2\sigma} [\square \sigma + (\partial \sigma)^2] \\ &\rightarrow -\frac{3}{4\kappa} \int d^4x \left[\beta_2^2 (\partial \sigma)^2 e^{\beta_2 \sigma} - \frac{72\pi^2}{\kappa} f(Q^2) e^{2\beta_2 \sigma} \right]. \end{aligned} \quad (7)$$

The integrands in these expressions have engineering dimensions 0 and 2 respectively, and so they must be multiplied by powers of e^σ to give a scalar density with total conformal weight 4. In the last forms of (6) and (7) we have allowed for the possibility that these terms are gravitationally dressed and that the classical scaling codimensions $(\beta_0)_d = 4$ and $(\beta_2)_d = 2$ are modified at the quantum level. For the Einstein term involving two derivatives of the σ field there is also the possibility of renormalization group mixing with operators of lower dimension, which is represented by the $f(Q^2)$ term in eq. (7). Indeed, a covariant computation of the renormalization of these operators due to the loop effects of

the free \square^2 propagator shows that β_0 and β_2 satisfy the equations,

$$\beta_0 = 4 + \frac{\beta_0^2}{2Q^2}, \quad (8)$$

and

$$\beta_2 = 2 + \frac{\beta_2^2}{2Q^2}, \quad (9)$$

while the operator mixing is given by

$$f(Q^2) = \frac{\alpha^2}{Q^2} \left[1 + \frac{4\alpha^2}{Q^2} + \frac{6\alpha^4}{Q^4} \right], \quad (10)$$

and $\alpha \equiv \beta_2/2$ in the present notation [1]. The classical values for the codimensions β_0 and β_2 are recovered only in the limit $Q^2 \rightarrow \infty$, where quantum fluctuations of the conformal factor are suppressed. Notice that this classical limit is the analog of the opposite limit of the central charge (*i.e.* $c \rightarrow -\infty$) in the 2D case, since Q^2 is positive for free conformal matter fields.

Corroboration of the anomalous scaling dimensions (8) and (9) comes from the canonical quantization of the σ theory on the Einstein space $R \times S^3$ where an infinite tower of discrete diffeomorphic invariant states, labelled by the integers, was obtained [5]. In the semi-classical limit $Q^2 \rightarrow \infty$, these states are created by operators which are volume integrals of integer powers of the Ricci scalar, *i.e.*,

$$\int d^4x \sqrt{g} R^n \sim \int d^4x e^{\beta_{2n}\sigma} \left[(\partial\sigma)^{2n} + \dots \right], \quad (11)$$

where

$$\beta_{2n} = 4 - 2n + \frac{\beta_{2n}^2}{2Q^2}. \quad (12)$$

The ellipsis in (11) refers to the operators with lower numbers of derivatives which mix with $(\partial\sigma)^{2n}$ under the renormalization group at finite Q^2 , analogous to (7) and (10) for $n = 1$. The exact forms of this operator mixing is not determined by the canonical analysis, but the values of all the β_{2n} are fixed by the Hamiltonian state condition (the analog of the $L_0 - 1$

condition on the physical states in $D = 2$). The first two of the discrete physical states are created by operators of the form (6) and (7) and the values of the scaling exponents are identical to (8) and (9) obtained in the covariant Euclidean approach.

Now the volume term evaluated on the spike configuration behaves like

$$S_0[\sigma_S] \sim -\frac{a^4}{(4 + \beta_0 q)} \left[c_0 - \left(\frac{L}{a} \right)^{4 + \beta_0 q} \right] = \frac{2a^4 Q^2}{\beta_0^2} \left[c_0 - \left(\frac{L}{a} \right)^{-\frac{\beta_0^2}{2Q^2}} \right], \quad (13)$$

for $L \gg a$ and $q = -1$, which is the only value for which the integral is IR convergent for any positive value of Q^2 (for which $\text{Re } \beta_0 > 4$). In this expression c_0 is a positive constant of order unity whose value depends on the precise way the UV cut-off a is introduced. In a similar manner, the Einstein term evaluated on the spike configuration behaves like

$$S_2[\sigma_S] \sim \frac{q^2 a^2}{(2 + \beta_2 q)} \left[c_2 - \left(\frac{L}{a} \right)^{2 + \beta_2 q} \right] = -\frac{2a^2 Q^2}{\beta_2^2} \left[c_2 - \left(\frac{L}{a} \right)^{-\frac{\beta_2^2}{2Q^2}} \right] \quad (14)$$

for $L \gg a$ and $q = -1$, where c_2 is another constant (dependent on a^2/κ). Inspection of the previous form shows that $q = -1$ is again the only value of q for which the Einstein action evaluated on the spike configuration is convergent as $L \rightarrow \infty$, for any positive value of Q^2 (for which $\text{Re } \beta_2 > 2$).

Hence, knowledge of the scaling exponents of the Einstein and volume operators allows us to compute their values on the singular spike configurations and show that they are subdominant to S_E in the infinite volume continuum limit for all sufficiently large and positive Q^2 , if and only if $q = -1$. With $q = -1$ selected in this way, we obtain from our previous evaluation of the free energy of the spike (5) the critical value,

$$Q_{cr}^2 = 8. \quad (15)$$

For $Q^2 > 8$ the solution of the quadratic relations (9) and (8), *viz.*,

$$\beta_0 = Q^2 \left(1 - \sqrt{1 - \frac{8}{Q^2}} \right) \quad (16)$$

and

$$\beta_2 = Q^2 \left(1 - \sqrt{1 - \frac{4}{Q^2}} \right) \quad (17)$$

are indeed both real and $\text{Re } \beta_0 > 4$, $\text{Re } \beta_2 > 2$ for $8 < Q^2 < \infty$, so that the previous argument for the irrelevance of the Einstein and cosmological terms in the infinite volume limit are justified *a posteriori*. We have chosen the minus sign for the square roots in (16) and (17) in order that $\beta_{2n} \rightarrow (\beta_{2n})_{cl} = 4 - 2n$ in the semi-classical limit $Q^2 \rightarrow \infty$.

When $Q^2 < 8$, then one of these exponents (namely β_0) becomes complex and we should expect qualitatively different behavior of the theory. This is an independent indication of the critical value $Q_{cr}^2 = 8$. From the simple BKT-like argument, we expect that $Q^2 < 8$ corresponds to a phase which is dominated by a dense gas of $q = -1$ spikes. The metric of the singular $q = -1$ configuration (with the origin, $x_0 = 0$) is

$$ds^2 = e^{2\sigma_S} d\bar{s}^2 = \frac{(dr^2 + r^2 d\Omega^2)}{r^2} = (d \ln r)^2 + d\Omega^2, \quad (18)$$

which is just that of $R \times S^3$, if we start with the flat R^4 metric background, $d\bar{s}^2$. Since the S^3 has arbitrary radius and the R is the entire real line, this geometry is arbitrarily elongated and thin. Since any region is locally like that of R^4 , a “gas” of such configurations means that these “spikes” (or perhaps more appropriately, “tubes” or “punctures”) will look like a large number of branches with a hierarchy of thinner and thinner sub-branches. A random geometry with many such spiky extrusions looks very much like the elongated phase described in ref. [7]. Conversely, if $Q^2 > 8$ then the scaling dimensions (16) and (17) are real, the spikes are suppressed and one would expect to find that the partition function is dominated by much smoother configurations, which at very large Q^2 are well-approximated by semi-classical metrics.

Thus, $Q_{cr}^2 = 8$ behaves in many respects like the $c = 1$ case in 2D gravity, with the important difference that the addition of conformal matter fields brings us into the smooth phase $Q^2 > 8$ rather than into the elongated or branched polymer phase $c > 1$ (see eq. (2)).

Now in pure simplicial gravity, *i.e.* without the introduction of matter, only an elongated phase and a crumpled phase (where the geometries collapse upon themselves) have been found. There is a transition between them as the lattice coupling corresponding to the Einstein term is varied, although apparently there is as yet no universal agreement as to the order of this transition. From the point of view of the considerations presented here the numerical results are consistent with the hypothesis that pure gravity has $Q^2 = Q_{grav}^2 \lesssim 8$, and only a spiky elongated phase in the continuum limit. We should emphasize in this connection that the anomaly generated action S_E is positive definite for $Q^2 > 0$ so that there is no conformal factor problem, nor any need for a conformal rotation as in the Einstein theory. Hence, the statistical continuum theory of random four-geometries described by (1) is well-defined for $0 < Q^2 < 8$ even if these geometries turn out to be very jagged.

The main theoretical difficulty in determining Q_{grav}^2 is that the Einstein theory is neither conformally invariant nor free, so that a method for evaluating the strong infrared effects of spin-2 gravitons in the continuum must be found that is insensitive to ultraviolet physics. In ref. [2] we performed a strictly perturbative computation which gives the value $Q_{grav}^2 = 1411/180 \approx 7.9$ for the graviton contribution. Since the method used is based on the heat kernel expansion, there is the possibility of mixing up ultraviolet with infrared effects in this evaluation, and we cannot regard it as definitive. However, a computation using the totally different conformally invariant Weyl tensor-squared action leads to a similar value $Q_{grav}^2 = 8.7$. Hence it is likely that the correct infrared graviton contribution to Q^2 is in the neighborhood of 8, and that most of this large value is due to the spin of the field, which is larger than any other individual field's contribution of lower spin. If the value of Q^2 in the pure gravity theory turns out to be less than eight, then one would not expect to find a second order phase transition to the continuum limit in the simplicial simulations with geometries that are smooth. Instead the continuum limit of the random geometries would be an elongated phase, filled with many spiky extrusions.

If this is indeed the correct interpretation of the numerical results to date, then the introduction of some number of free conformal matter fields to the simplicial simulations would push the value of Q^2 above 8 and lead to a continuum limit exhibiting a smooth phase, perhaps by softening a weakly first order transition between the elongated and crumpled phases into a second order transition with the new phase appearing in the phase diagram extended into the $Q^2 > Q_{grav}^2$ direction. Taking the 1411/180 number at face value and using the known contributions to Q^2 for lower spin fields in eq. (2), one finds that it would take the introduction of 57 conformal scalars, but *only one photon* to induce this transition to the smooth phase.

Scaling relations. Assuming that the smooth phase with $Q^2 > 8$ exists, the exponents make definite predictions for the scaling of the fixed volume partition function,

$$Z(\kappa, \lambda) \equiv \int [\mathcal{D}\sigma] \exp \left(-S_E[\sigma] - \frac{1}{2\kappa} S_2[\sigma] - \lambda S_0[\sigma] \right). \quad (19)$$

The covariant continuum measure $[\mathcal{D}\sigma]$ has been discussed in several previous articles [2, 12]. By inserting a delta function of physical four-volume, we obtain the fixed volume partition function

$$Z(\kappa; V) \equiv \int [\mathcal{D}\sigma] \exp \left(-S_E[\sigma] - \frac{1}{2\kappa} S_2[\sigma] \right) \delta(S_0[\sigma] - V). \quad (20)$$

If the effective action appearing here, namely the sum of the Einstein and anomaly induced terms, is written in an arbitrary curved background and one studies the effect on the finite volume partition function of a shift in σ ,

$$\sigma \rightarrow \sigma + \omega \quad (21)$$

then one finds that

$$Z(\kappa; V) = \exp \left[-\omega \left(\beta_0 + Q^2 \chi_E \right) \right] Z(\kappa e^{-\omega \beta_2}; e^{-\omega \beta_0} V), \quad (22)$$

where the Euler number $\chi_E = 2$ for fixed S^4 topology. Since σ has been integrated out, the resulting $Z(\kappa; V)$ must be independent of the shift (21), which is only consistent with (22) if the fixed volume partition function can be expressed in the form,

$$Z(\kappa; V) = V^{-1-2\frac{Q^2}{\beta_0}} \tilde{Z}(\kappa V^{-\frac{\beta_2}{\beta_0}}), \quad (23)$$

for some function \tilde{Z} of a single argument. This scaling relation differs from that obtained in eq. (10) in our previous letter [4], since now we have allowed the volume and Einstein terms (6) and (7) to have independent scaling exponents in agreement with our subsequent canonical analysis [5] and ref. [13].

By using the relation (16) we now obtain the susceptibility exponent

$$\gamma(Q^2) = 2 - 2\frac{Q^2}{\beta_0} = -2\frac{\sqrt{1 - \frac{8}{Q^2}}}{1 - \sqrt{1 - \frac{8}{Q^2}}} \quad (24)$$

instead of eq. (23) of ref. [4]. Notice that if Q^2 is close to 8 then the susceptibility exponent is close to zero, which is what has been found in the simulations to date. As shown in Fig. 1, γ is negative for $Q^2 > 8$ and approaches zero from below as $Q^2 \rightarrow 8$.

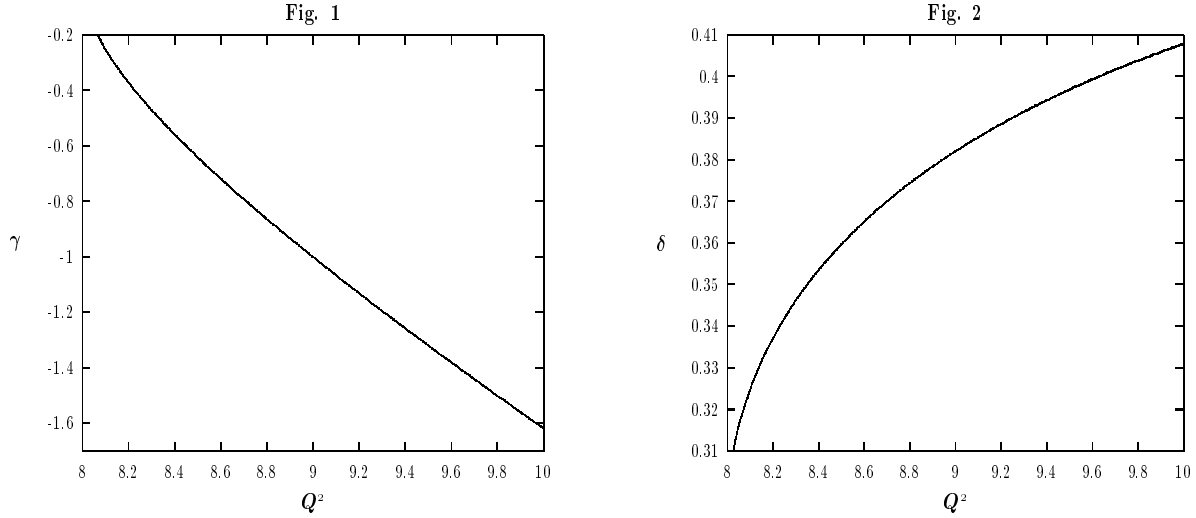
The form (23) also implies that we should require $\kappa V^{-\frac{\beta_2}{\beta_0}}$ to remain finite in the scaling limit, or equivalently,

$$\kappa \sim V^\delta \quad \text{as} \quad V \rightarrow \infty, \quad (25)$$

with

$$\delta = \frac{\beta_2}{\beta_0} = \frac{1 - \sqrt{1 - \frac{4}{Q^2}}}{1 - \sqrt{1 - \frac{8}{Q^2}}}. \quad (26)$$

We remark that this relation differs from eq. (22) of ref. [4] because of the addition of the second independent cosmological operator which we did not consider in that work. The exponent δ is plotted as a function of Q^2 in Fig. 2. For Q^2 near 8, $\delta \simeq 0.3$, which differs from the value 0.47 ± 0.03 reported in the simulations of ref. [7]. However, if our hypothesis is correct, namely that Q^2 for pure gravity is less than 8, then the smooth phase



Figures 1 and 2. The susceptibility and scaling exponents γ and δ as functions of the anomaly coefficient Q^2 .

where eq. (26) applies has not yet been produced in the simulations. Of course, when the numerical situation has been clarified and an unambiguous continuum limit with a smooth phase has been demonstrated, then there is only one free parameter and both (24) and (26) must be consistent with the same value for Q^2 , if the conformal fixed point relations predicted by the continuum theory are correct.

Finally, one can consider any operator composed of either metric or matter fields with scaling dimension $\bar{\Delta}$, in the absence of dressing. By inserting this operator in the fixed volume partition function

$$\langle \mathcal{O}_{\bar{\Delta}} \rangle_V = \frac{1}{Z(\kappa; V)} \int [\mathcal{D}\sigma] \mathcal{O}_{\bar{\Delta}} \exp \left(-S_E[\sigma] - \frac{1}{2\kappa} S_2[\sigma] \right) \delta(S_0[\sigma] - V) , \quad (27)$$

and repeating the shift (21) and scaling argument for this quantity, we find

$$\langle \mathcal{O}_{\bar{\Delta}} \rangle_V \sim V^{1-\frac{\bar{\Delta}}{4}} , \quad (28)$$

where the full scaling dimension Δ is related to the “classical” dimension $\bar{\Delta}$ by

$$\Delta = 4 - 4 \frac{\beta_{\bar{\Delta}}}{\beta_0} = 4 \frac{\sqrt{1 - 2 \frac{(4-\bar{\Delta})}{Q^2}} - \sqrt{1 - \frac{8}{Q^2}}}{1 - \sqrt{1 - \frac{8}{Q^2}}} . \quad (29)$$

Here $\beta_{\bar{\Delta}}$ is the relevant codimension of the gravitational dressing determined by

$$\beta_{\bar{\Delta}} = 4 - \bar{\Delta} + \frac{\beta_{\bar{\Delta}}^2}{2Q^2}. \quad (30)$$

At large Q^2 one has:

$$\Delta - \bar{\Delta} = \frac{\beta_{\bar{\Delta}}^2}{32Q^2} \Delta(4 - \Delta) = \frac{1}{2Q^2} \bar{\Delta}(4 - \bar{\Delta}) + \dots \quad (31)$$

For the volume and Einstein operators for which $\bar{\Delta}$ is 0 and 2, respectively, we recover the previous relations (8) and (9).

If these scaling relations can be verified in the smooth phase, and the infrared conformal fixed point predicted by the action (1) confirmed, then the lattice could provide a nonperturbative method for measuring the contribution Q_{grav}^2 . In addition to being interesting in its own right by exploring a nontrivial fixed point of 4D quantum gravity, the scaling relations could also find applications in cosmology [14].

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